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**ATTITUDE CONTROL OF  
SPACE VEHICLES**

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## **PREFACE**

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## ATTITUDE CONTROL OF SPACE VEHICLES

## ABSTRACT

C. R. Gates

The problem of sensing and controlling the angular orientation of a space vehicle is discussed. Optical, radio, and inertial methods of angle-sensing are described, and application of control torques by reaction jets, flywheels, radiation pressure, and gravitational gradient are compared. Requirements for angular accuracy likely to be imposed by maneuvering, performing measurements for navigation, making scientific measurements, radio communication, and solar cell orientation are described. Attitude control requirements and techniques for Earth satellites and deep-space probes are compared.

## I. INTRODUCTION

This paper discusses the problem of attitude control of space vehicles. In particular, we refer to sensing and controlling the angular orientation of a space vehicle after the vehicle has been injected into orbit, and is in free fall.

The paper is predominantly tutorial in character, and comprehensive system designs are not given.

## II. FUNCTIONS AND ACCURACIES

In order to establish a frame work for subsequent discussion, we shall examine some of the reasons for controlling attitude. From these reasons we hope to deduce accuracy or other functional requirements. Table I gives the best-known reasons for attitude control together with estimates of accuracies needed.

TABLE I  
ATTITUDE CONTROL FUNCTIONS AND ACCURACIES

Function	Accuracy
A. Solar Energy Collection	1° to 10°
B. Observation	.1" to 1°
C. Communication	1° to 5°
D. Navigation	5" to 1°
E. Maneuvering	1/4° to 1°
F. Environmental Control	1° to 10°

The numbers in Table I are the author's own, and are intended to be suggestive rather than definitive. A few remarks about the numbers are in order. For solar energy collection (A), we observe that solar cells suffer a cosine loss, and  $5^\circ$  to  $10^\circ$  pointing error should be tolerable. For a solar furnace which focuses the energy an accuracy of  $1^\circ$  or perhaps better would be needed. Under observation (B), the most demanding requirement ever suggested to the author concerned an orbiting observatory in which a telescope, rigidly attached to the hull of the vehicle, was to make extended observations, which required an accuracy of  $.1''$ ; most observations require at least  $1^\circ$  accuracy. For communication (C), a figure of  $1^\circ$  seems compatible with the vehicle antenna gains we are likely to possess for some time. For navigation (D), we refer to the accuracy with which the vehicle must be stabilized during a navigational sighting of the stars and planets. The accuracy requirement depends heavily upon the method of measurement. For maneuvering (E), which describes relatively small periods of thrust as may be necessary to correct the trajectory, an accuracy of no better than  $1/4^\circ$  appears adequate. And finally, under environmental control (F), which is most likely to be temperature control, accuracies comparable to (A) are sufficient.

It is interesting to note that in Table I a total of six reasons are given for requiring attitude control, and there may well be others that have been overlooked. The vehicle has only three degrees of rotational freedom, and in particular, if one aspect of the vehicle be pointed in a given direction, say at the Sun, we have no assurance that a second aspect of the vehicle can be made to point in another given direction, say at the Earth, even if the vehicle be rolled about the vehicle-Sun line. Thus we immediately encounter the

"degrees-of-freedom" problem which must plague us for all time to come with systems of hinges, universal joints, swivels, gimbals, etc. Imagine, for example, the circumstance in which we approach a planet and wish to simultaneously point solar cells at the Sun, an antenna toward the Earth, a camera at the target planet, sextants at the planet and one or two stars, and a maneuvering rocket in some other arbitrary direction.

### III. ATTITUDE SENSING

Before proceeding to a consideration of torques for attitude control, we will first treat attitude sensing. Conceptually, vehicle attitude may be sensed from within the vehicle or externally from the Earth. We shall consider only the more important case of vehicle-contained attitude sensing.

Our physicist colleagues inform us that there are two types of fields, gravitational and electromagnetic. Attitude sensing (and control) schemes have been proposed which depend on either or both. For this discussion we shall subdivide further and treat (a) gravitational, (b) radio, and (c) optical sensing.

For gravitational sensing we observe that under the influence of a central inverse-square force field, the force throughout a finite body is not uniform in either direction or magnitude. Thus in a rigid body only the point at the center of gravity<sup>(1)</sup> behaves as if it were acted upon only by the gravitational force. All other points in the body are acted upon by internal forces as well as by gravity.

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(1) Actually the apparent center of gravity is shifted slightly toward the center of the force field because of the non-uniformity of the field; however, the effect is small.

These forces can be described by letting  $\bar{G}(\bar{r})$  denote the gravity field, where  $\bar{r}$  is the radius vector from the center of the field. Let  $\bar{r}_0$  be a vector to the center of gravity of the vehicle and  $\bar{r}_1$  be a vector to any other point in the body. The internal force/mass is given by (see Fig. 1)

$$\bar{F}(\bar{r}_1) = \bar{G}(\bar{r}_1) - \bar{G}(\bar{r}_0) \quad (1)$$

It may be possible to measure this force and thereby determine the direction of and/or distance to the attracting body. To explore further, write  $G(r) = -\frac{KM}{r^3}$ , where  $K$  is the gravitational constant, and  $M$  the mass of the attracting body. Further, construct a (two-dimensional) rectangular coordinate system with its origin at the center of gravity of the vehicle and with the negative y-axis, at the instant of interest, at the center of the force-field. We then find that, to first order,

$$\bar{F} = \frac{KM}{r_0^3}(\bar{i} x - \bar{j} 2y) \quad (2)$$

The convention here is that the units of  $\bar{F}$  indicate acceleration. The sign convention is that a point at  $(x = 0, y = y_1)$  is being accelerated "upward," and hence is acted upon by a force in the negative y direction. Figure 3 shows  $\bar{F}$  on the periphery of a spherical vehicle.

We first observe that, due to the  $1/r_0^3$  term, the force at great distance must be very small. Indeed at  $10^6$  miles from the Earth, an acceleration one foot in the y-direction from the CG is only  $6.4 \times 10^{-15}g$  (see Fig. 2). Also,  $\bar{F}$  is zero only at the CG of the body, and therefore we could not determine the direction of  $\bar{G}$  by a nulling technique. It appears unlikely, then, that on an interplanetary



journey we would be able to determine direction or distance by this method, unless very near to an attracting body. <sup>(1)</sup> Note further that the angular rates of the vehicle would have to be held to a low value in order to avoid centrifugal forces. For the example given previously, where the radius arm was one foot and the acceleration was  $6.4 \times 10^{-15} g$ , an angular velocity given by  $6.4 \times 10^{-15} = \omega^2 R/g$ , where  $R = 1$ , gives  $\omega = 4.5 \times 10^{-7}$  rad/sec, or about .1°/hr as that angular velocity where centrifugal and gravitational gradient forces are equal (this occurs at a distance of one million miles from the Earth).

Near a force center the gravitational gradient should have more utility. Consider the interesting case of a circular near-Earth satellite, in which the vehicle is to be pointed at the Earth. Since the coordinate system now rotates, we must consider the centrifugal force. For a circular satellite we have  $\frac{KM}{r_o^2} = \omega^2 r_o$ , where  $r_o$  is the height from the center of the Earth, and  $\omega$  is the angular velocity. The centrifugal acceleration at a point within the body is  $\omega^2(\bar{i} \times + \bar{j} y)$ , using our previous coordinate system. Thus the total force/mass is given by

$$\begin{aligned}\bar{F}_T &= \frac{KM}{r_o^3}(\bar{i} \times -\bar{j} 2y) - (\bar{i} \times + \bar{j} y) \\ &= \frac{KM}{r_o^3}(-\bar{j} 3y)\end{aligned}\tag{3}$$

<sup>(1)</sup>Note that since  $\bar{F}$  contains a term  $M/r^3$  the differential force at the surface of a body depends on its density; thus  $\bar{F}$  is greater at the surface of the Earth than at the surface of the Sun, the Sun being less dense.

Thus all of the force is directed along the radius from the center of the Earth. In a satellite at an attitude of 300 miles the force/mass at a distance one foot away from the CG (along the radius vector) is about  $10^{-7}G$ . It would seem feasible to measure this force; however, as will be pointed out in Part IV, it may be more desirable to combine attitude sensing and control into a single system.

In the other two schemes for attitude sensing, radio and optical, we are on more familiar ground. Vehicle-contained radio sensing would require a space-borne tracking radar and a dependable source of radio energy from the Earth. It has the advantage of control and encoding of the radio link, which link may be required for other purposes of telemetry, command, etc. Thus the vehicle is unlikely to confuse the Earth with a star or another planet. The disadvantages, however, are great. The vehicle-borne equipment may be complex, and we may have to provide many transmitters on the Earth. We are restricted to knowing only the direction of the Earth, as it seems difficult to locate transmitters elsewhere; thus, attitude data concerning the angle about the Earth-vehicle line are unavailable. Also, while the accuracies attainable, a few milliradians, appear adequate for all but navigational purposes, optical systems have a greater potential.

It is optical attitude sensing systems which have by far the greatest promise. Sensors are of reasonable size, weight, and power consumption. For Earth satellites we may expect IR sensors detecting the horizon; for lunar probes, a system using the Sun, the Earth and/or the Moon appears most practical; and for deep space probes we may expect sensing of the Sun, planets, and stars. Accuracies of a few seconds or tens of seconds of arc for star sensors appear

feasible. An advantage of optical attitude sensing, the multiplicity of available observable bodies, is at the same time a disadvantage, presenting problems in identification. We may expect this problem to be mollified (1) by use of the Sun as one reference, and (2) by use of the spectra and magnitudes of stars as an aid in identification. It should be possible to identify the Earth by aligning one axis of the vehicle toward the Sun, and searching, at a prescribed Earth-vehicle-Sun angle, by rolling about the Sun line (see Fig. 4). A radio link could then serve as a check.

#### IV. ATTITUDE CONTROL TORQUES

In providing control torques we may attempt to use some aspect of the natural environment, such as the gravitational gradient or solar radiation pressure or, we may use a self-generated torque, in which the system may be mass-conservative, using flywheels, or non-mass-conservative, using reaction jets. More than likely some combination of these techniques will be the most effective.

We observe that there are at least three distinct portions of flight when control must operate. These are (1), just following separation from the launch vehicle, when angular velocities may be large, (2), during trajectory-correcting maneuvers when disturbances may be large, and (3) during coasting. We shall principally treat (3), coasting.

Reaction jets offer a simple and well-understood source of torque. Requirements for gas are surprisingly low. For example, using a typical cold-gas system with a payload of several hundred pounds, we have the capacity for

about one million cycles to  $\pm 1^\circ$ /hour per pound of gas. Thus, even a trip of several months duration might require only 2-3% of the payload weight as gas. A principal problem of concern is leakage of the gas over such a long interval.

Flywheels are more interesting, and have the advantage of conceptually indefinite operation. However, in the presence of a steady disturbing torque the flywheel speed must build up, and a means of draining away the steady angular impulse must be present. Reaction jets, or possibly solar pressure panels, if adjustable, could serve for periodic zero-setting. Care in the design must be exercised to account for gyroscopic interaction between wheels, although a single spherical wheel may reduce or eliminate this problem. A flywheel which is 1% of the mass of the vehicle and which can operate at several thousand RPM should provide angular velocity corrections of several degrees per second.

Solar radiation pressure, at the distance of the Earth, is about  $5 \times 10^{-5}$  dynes/cm<sup>2</sup> for an absorbent surface, and  $10^{-4}$  dynes/cm<sup>2</sup> for a reflecting surface. Assuming an absorbing surface such as a panel of solar batteries, an area of  $10^3$  cm<sup>2</sup>, a lever arm of 2 ft, and a payload for which the moment of inertia of interest is 7 slug ft<sup>2</sup>, we achieve an angular acceleration of about  $3.2 \times 10^{-8}$  rad/sec<sup>2</sup>. Such an acceleration would result in an angular velocity of about .2°/sec after one day. This torque thus would probably not be useful as a primary control, but it might be effective in draining away angular impulse from a flywheel system.

Gravitational gradients, as noted in Part III, are useful only very near an attracting body. Indeed, an attitude control system designed for a deep-space

vehicle would be overtaxed near a planet. As may be seen from Eq. (3), the gravitational gradient provides a torque which will cause an asymmetric body to align its principal axis of inertia along the local vertical (see Fig. 5). If suitable damping is provided, a vertical seeking system results. Also, it may be observed that in the rotating coordinate system of an Earth pointing vehicle a Coriolis force is available; thus a gyro may be used to locate the direction of the angular velocity vector, and hence align the third axis of the vehicle.

## V. SPECIAL CONTROL PROBLEMS

In examining the attitude control problem from a control engineer's point of view we observe several departures from familiar practice: (a) The time-constants of operation are long, perhaps hours or days. (b) External damping is absent. (c) The dynamic range of operation is extreme. (d) Demands on reliability and lifetime are great.

In the analysis of the control loop items (a) and (b) above, long time-constants and zero damping, combine to place special attention on determination of angular rate. Rate determination to an accuracy of  $.1$  to  $.01^\circ/\text{hr}$  is desirable. With the sensors most often considered, analog differentiation of position seems unacceptable. Rate gyros would provide the desired accuracy, but they may have insufficient lifetime. It should be possible to determine rate optically, possibly with a interferometer; however, further development seems necessary. If we know the scale factor of the torquing system, differential rate is available, which should aid in the stabilization. Also, future digital control systems should

provide solutions. However, determination of rate appears to be a major current problem.

Under item (c), extreme dynamic range, we note that the attitude control system may be called upon to function from separation (from the launch vehicle) where separation disturbances may produce rates of several degrees/sec or perhaps 10,000°/hr, up to a mode of operation where rate limits of .1°/hr or less are desired. Thus, a dynamic range of  $10^5$  is possible.

Finally, under (d) reliability, we observe that attitude sensing and control equipment must be so designed that it can operate without the need for repair or adjustment for periods of months at a time.

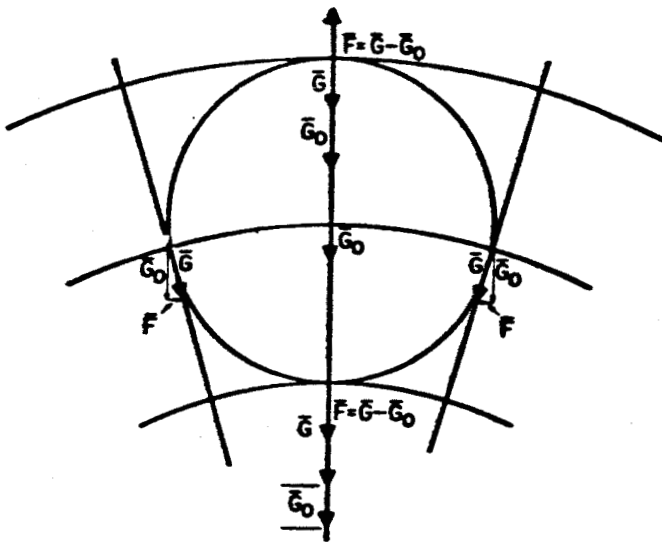


Fig. 1. Construction of Gravitational Gradient

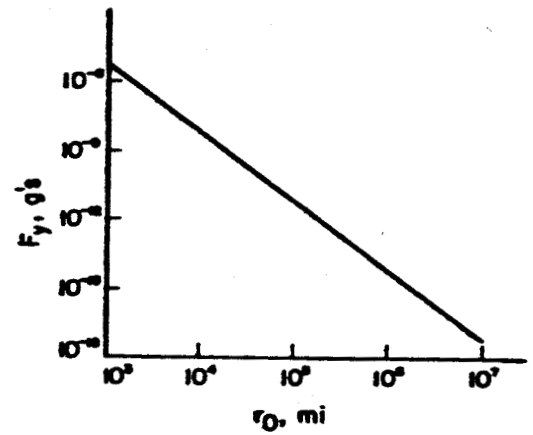


Fig. 2. Gravitational Gradient vs Distance from Earth

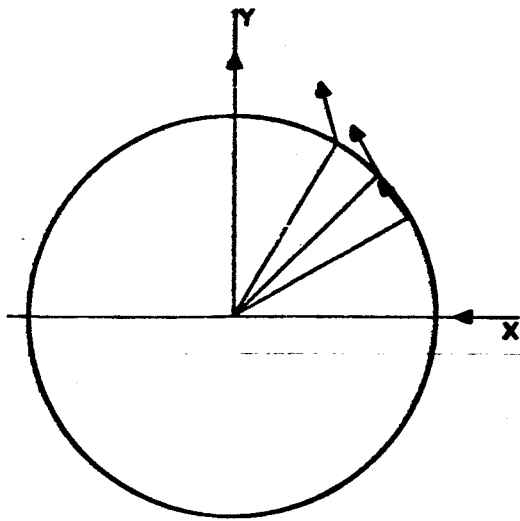


Fig. 3. Field of Gravitational Gradient

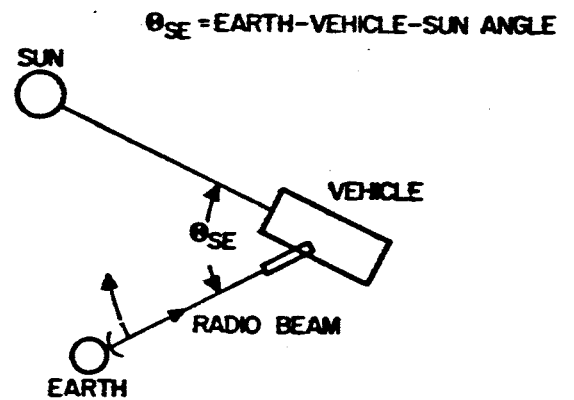


Fig. 4. Earth Identification

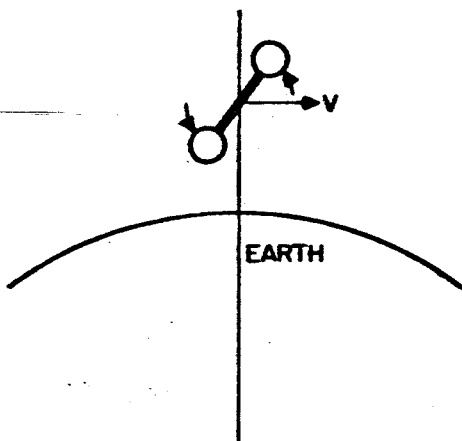


Fig. 5. Gravitational Gradient Torque